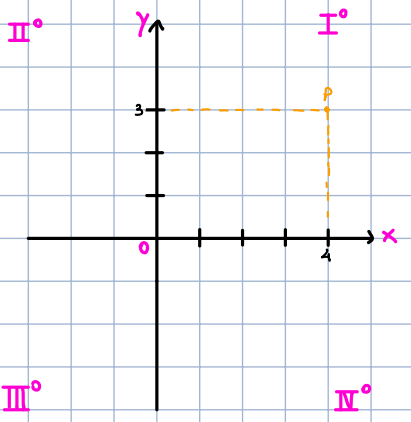
Piano Cartesiano

Un sistema di riferimento è una coppia di RETTE PERPENDICOLARI TRA LORO.

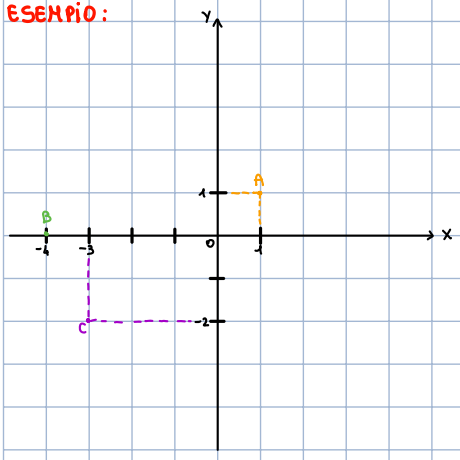


- asse verticale = ordinate (y)
- asse orizzontale = ascisse (x)
- il punto di incontro degli assi viene detto origine (O)
- Sono presenti 4 quadranti (I; II; III; IV)

ESEMPIO: P(4;3)

N.B. ogni punto del piano è identificato da una coppia di coordinate "$P(x_P; y_P)$"

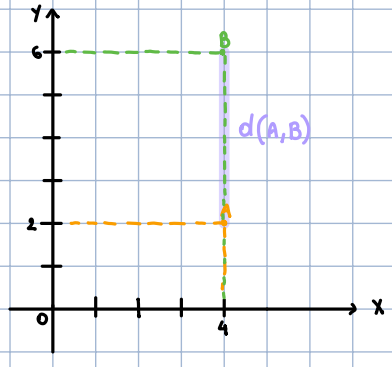
ESEMPIO:



A(1,1)
B(-4,0)
C(-3,-2)

Distanza tra 2 punti del piano

- i punti sono allineati verticalmente | orizzontalmente

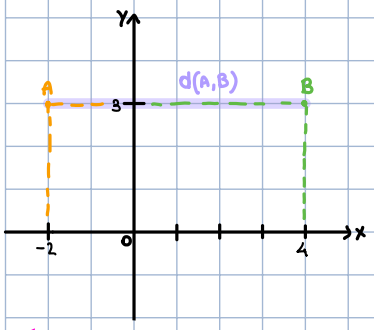


A(4,2)
B(4,6)

$d(A,B) = |y_A - y_B| =$
$|6 - 2| = 4$

valore assoluto

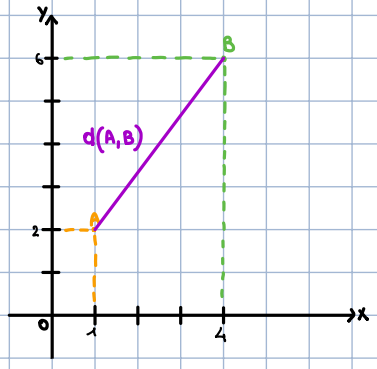
Si prende il numero senza segno



A(-2,3)
B(4,3)

$d(A,B) = |x_A - x_B| =$
$|-2-4| = 6$

◀ i due punti non sono allineati orizzontalmente | verticalmente



A(1,2)
B(4,6)
C(4,2)

Thm Pitagora: $i^2 = c_1^2 - c_1^2$

$\downarrow \sqrt{}$

$d(A,B) = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} =$

$\downarrow i$

$= \sqrt{(1-4)^2 + (2-6)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Retta nel piano

Ogni retta nel piano Cartesiano è caratterizzato da un'equazione che stabilisce la proprietà che devono soddisfare i punti che appartengono alla curva.

Eq. Retta: $y = mx + q$ (m e q numeri)

esempio

$y = 2x - 1$ $\quad m = 2 \quad q = -1$

$P(8,2) \in r$? ⟶ sostituisco $\quad 2 = 2 \cdot 8 - 1 \; ; \; 2 = 15$ ✗

$P(2,3) \in r$? ⟶ sostituisco $\quad 3 = 2 \cdot 2 - 1 \; ; \; 3 = 3$ ✓

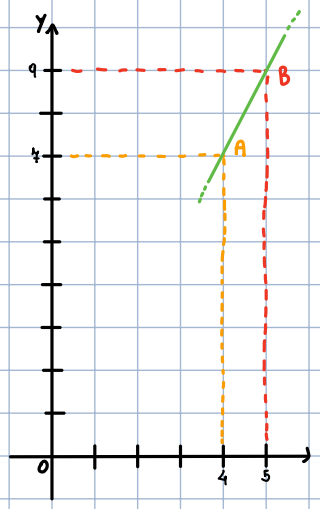
COME DISEGNARE UNA RETTA CONOSCENDO LA SUA EQUAZIONE

1 se voglio disegnare una retta mi occorrono 2 punti

$y = 2x - 1$ $x = 4$ $y = 7$ $A(4,7)$

$x = 5$ $y = 9$ $B(5,9)$

2 disegno i punti nel piano e traccio la retta



equazione della retta: $y = mx + q$

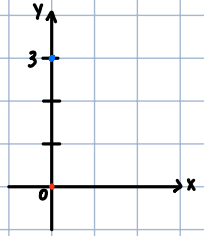
ANALISI DEI COEFFICIENTI m E q

q ⟶ intercetta con l'asse delle y

indica l'altezza a cui la retta intercetta l'asse delle y

ES.

- $y = -2x + 3$ ↓ q
- $y = 5x$ ↓ $q = 0$



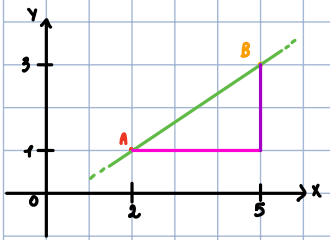
m = coefficiente angolare della retta

indica l'inclinazione della retta

ES.

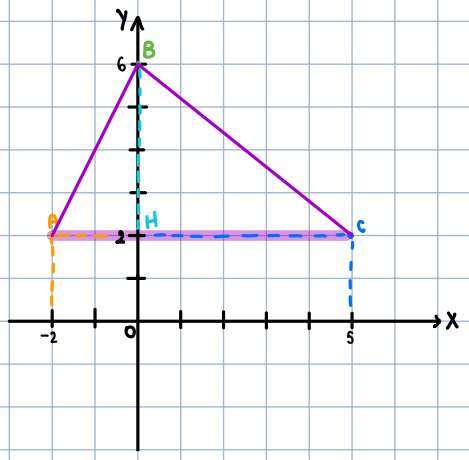
$$m = \frac{2}{4} = \frac{\text{variazione della y}}{\text{variazione della x}}$$

$$m = \frac{y_B - y_A}{x_B - x_A}$$



esercizi

Calcola l'area del triangolo che ha i vertici nei punti $A(-2,2), B(0,6), C(5,2)$



SVOLGIMENTO

$A = (b \cdot h) : 2$

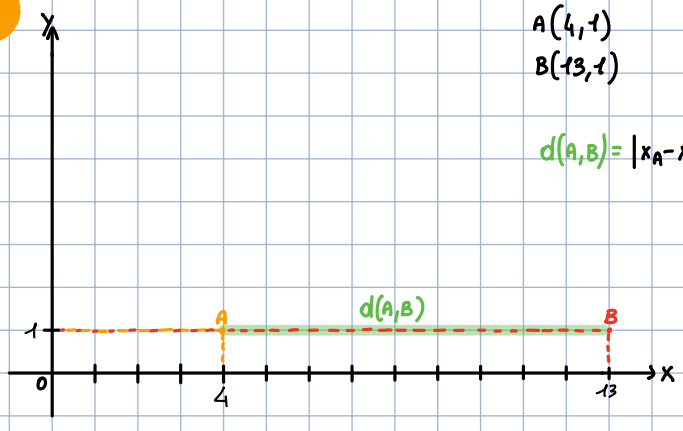
$b = \overline{AC} = |x_A - x_B| = |-2-5| = 7$

$h = \overline{BH} = |y_B - y_H| = |6-2| = 4$

$A = (7 \cdot 4) : 2 = 14$

24 $A(-7; 14), \quad B(1; 8).$

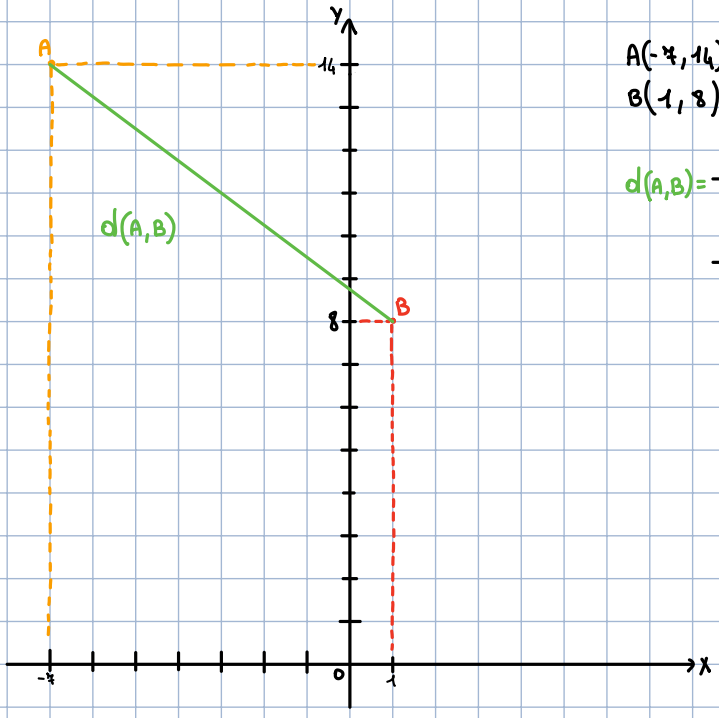
25 $A(4; 1), \quad B(13; 1).$



$A(4,1)$

$B(13,1)$

$d(A,B) = |x_A - x_B| = |13-4| = 9$



$A(-7,14)$

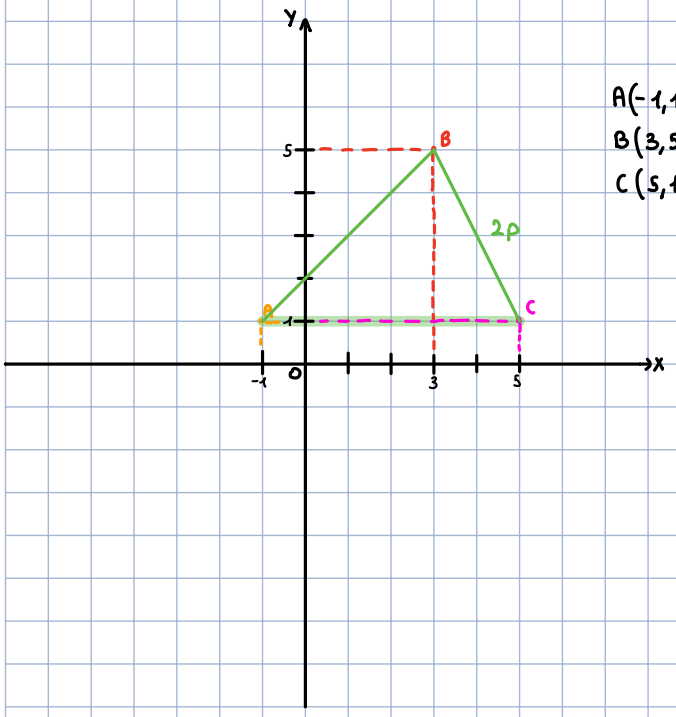
$B(1,8)$

$d(A,B) = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} =$

$\sqrt{(-7-1)^2 + (14-8)^2} =$

$\sqrt{64+36} = \sqrt{100} = 10$

73 $A(-1;1),\ B(3;5),\ C(5;1).$



$A(-1,1)$
$B(3,5)$
$C(5,1)$

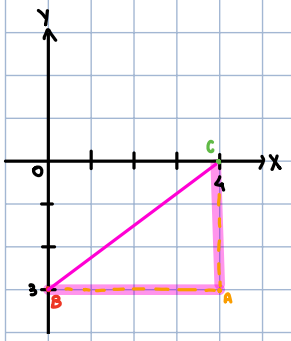
$2\rho = \overline{AB} + \overline{BC} + \overline{CA}$

$\overline{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} =$
$\sqrt{(-1-3)^2 + (1-5)^2} =$
$\sqrt{16+16} = \sqrt{32}$

$\overline{BC} = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2} =$
$\sqrt{(3-5)^2 + (5-1)^2} =$
$\sqrt{4+16} = \sqrt{20}$

$\overline{AC} = |x_A - x_C| = |-1-5| = 6$

$2\rho = 6 + \sqrt{32} + \sqrt{20}$
$6 + 5{,}6 + 4{,}5 = 16$?



$A(4,-3)$
$B(0,-3)$
$C(0,4)$

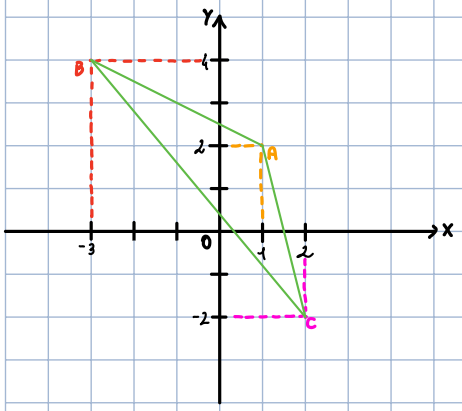
$\overline{BC} = \sqrt{(4-0)^2 + (0+3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\overline{AB} = |4-0| = 4$

$\overline{CA} = |4-3| = 3$?

$2\rho = l + l + l = 5+4+3 = 12$

$A = b \cdot h : 2 = 4 \cdot 3 : 2 = 6$



$A(1,2)$
$B(-3,4)$
$C(2,-2)$

$A = b \cdot h = 5 \cdot 4 = 35$

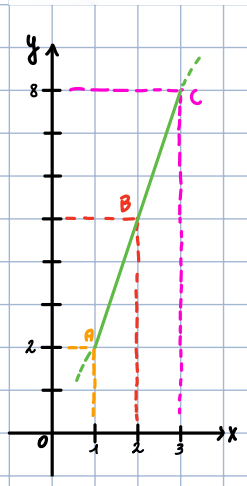
$A_{T_1} = \frac{5 \cdot 4}{2} = \frac{35}{2}$

$A_{T_2} = \frac{4 \cdot 2}{2} = 4$

$A_{TOT} = \frac{B + b \cdot h}{2} = \frac{4 + 2 \cdot 1}{2} = \frac{9}{2}$

$A_3 = 35 - \frac{35}{2} - 4 - \frac{9}{2} = \frac{18}{2} = 9$

135 $y = 3x - 1$



$y = mx + q$

A: $2 = 3(1) - 1$ $A(1,2)$

$2 = 2$

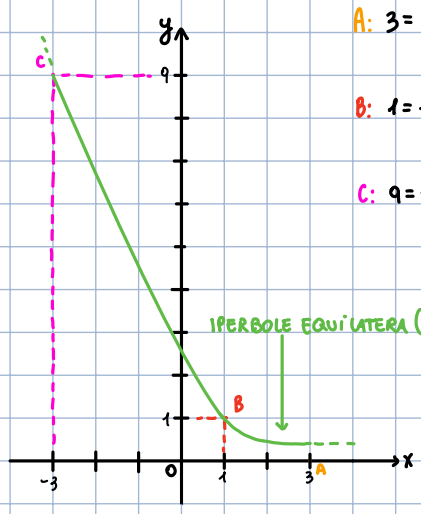
B: $5 = 3(2) - 1$ $B(2,5)$

$5 = 5$

C: $8 = 3(3) - 1$ $C(3,8)$

$8 = 8$

136 $y = -2x + 3$



A: $3 = -2(0) + 3$ $A(0,3)$

$3 = 3$

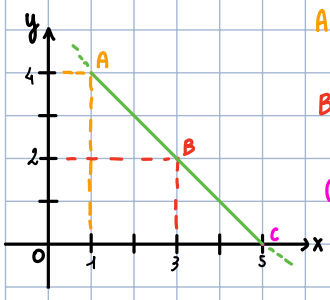
B: $1 = -2(1) + 3$ $B(1,1)$

$1 = 1$

C: $9 = -2(-3) + 3$ $C(-3,9)$

$9 = 9$

137 $y = -x + 5$



A: $4 = -(1) + 5$ $A(1,4)$

$4 = 4$

B: $2 = -(3) + 5$ $B(3,2)$

$2 = 2$

C: $5 = -(0) + 5$ $C(0,5)$

$5 = 5$

143 $A(1;4)$, $B\left(-\frac{1}{2};0\right)$, $C(-5;36)$; $y = 8x - 4$.

$4 = 8(1) - 4$

$4 = 8 - 4$

$4 = 4$ ✓

$0 = 8\left(-\frac{1}{2}\right) - 4$

$0 = -\frac{8}{2} - 4$

$0 = -4 - 4$

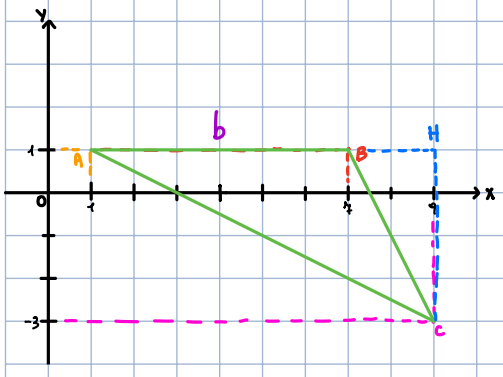
$0 = -8$ ✗

$36 = 8(-5) - 4$

$36 = -40 - 4$

$36 = 36$ ✓

91 $A(1;1)$, $B(7;1)$, $C(9;-3)$.

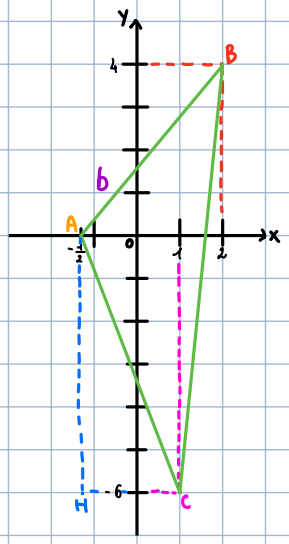


$b = |x_A - x_B| = |1 - 7| = 6$

$h = |y_C - y_H| = |-3 - 1| = 4$

$A_T = \frac{b \cdot h}{2} = \frac{6 \cdot 4}{2} = 12$

93 $A\left(-\frac{1}{2};0\right)$, $B(2;4)$, $C(1;-6)$.



$b = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} =$

$\sqrt{\left(-\frac{1}{2} - 2\right)^2 + (0 - 4)^2} =$

$\sqrt{\frac{25}{2} + 16} = \sqrt{\frac{57}{2}} = \sqrt{28,5} = 5,3$

$h = |y_A - y_H| = |0 - 6| = 6$

$A_T = \frac{b \cdot h}{2} = \frac{5,3 \cdot 6}{2} = 16$

138 $y = 2x - 4$

A: $-4 = 2(0) - 4$ $A(0,-4)$

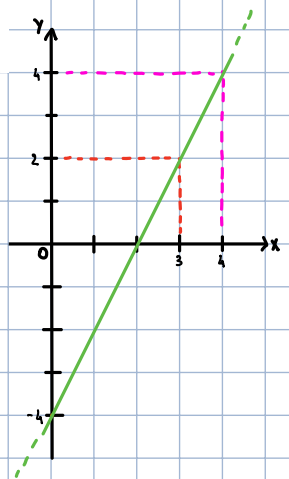
$-4 = -4$

B: $2 = 2(3) - 4$ $B(3,2)$

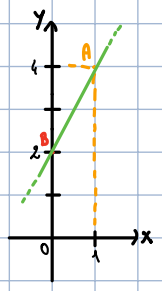
$2 = 2$

C: $4 = 2(4) - 4$ $C(4,4)$

$4 = 4$



ESERCIZIO n° 1



$y = 2x + 2$

$4 = 2(1) + 2$ $A(1,4)$

$4 = 4$

$2 = 2(0) + 2$ $B(0,2)$

$2 = 2$

$q = 2$

$$m = \frac{y_B - y_A}{x_B - x_A} = \frac{2-4}{0-1} = \frac{-2}{-1} = 2$$

ESERCIZIO n° 2

$V_m = 3\ m/s$

$d(S_0) = 1\ m$

$S = S_0 + Vt \rightarrow y = mx + q$

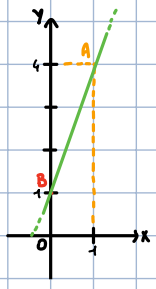
$S = 1 + 3t$ $\quad y = 3x + 1$

$4 = 3(1) + 1$ $A(1,4)$

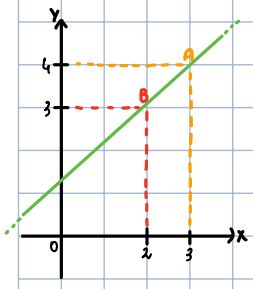
$4 = 4$

$1 = 3(0) + 1$ $B(0,1)$

$1 = 1$



ESERCIZIO n° 3

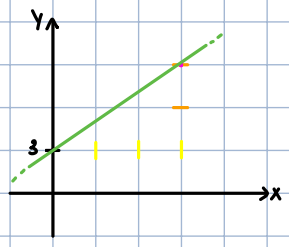


$$m = \frac{y_B - y_A}{x_B - x_A} = \frac{3-4}{2-3} = \frac{-1}{-1} = 1$$

$A(3,4)$

$B(2,3)$

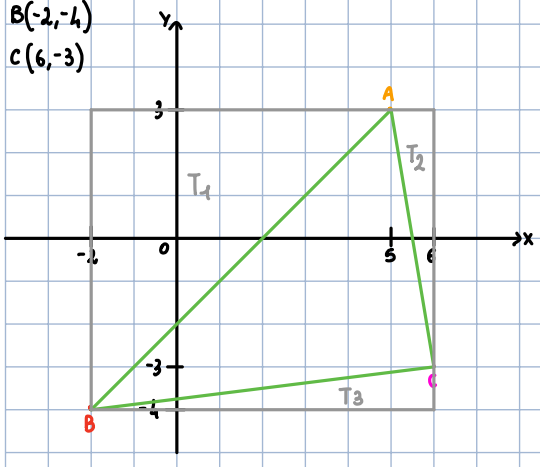
$y = \frac{2}{3}x + 3$



$A(5,3)$

$B(-2,-4)$

$C(6,-3)$



$A_R = 7 \cdot 8 = 56$

$A_{T1} = \frac{b \cdot h}{2} = \frac{7 \cdot 7}{2} = \frac{49}{2} = 24{,}5\ cm^2$

$A_{T2} = \frac{7 \cdot 1}{2} = \frac{7}{2} = 3{,}5\ cm^2$

$A_{T3} = \frac{8 \cdot 1}{2} = \frac{8}{2} = 4\ cm^2$

$A_{TOT} = 24{,}5 - 3{,}5 - 4 = 17\ cm^2$